16: Statistical models for wind data analysis:

The Weibull and Rayleigh distributions can be used to describe the wind variations in a regime with an acceptable accuracy level.

16.1: Weibull distribution: the variations in wind velocity are characterized by the two functions; (1) The probability density function and (2) The cumulative distribution function. The probability density function **(f (V))** indicates the fraction of time (or probability) for which the wind is at a given velocity (**V**). It is given by

$$\mathbf{f(V)} = \frac{\mathbf{K}}{\mathbf{c}}\left[\frac{\mathbf{V}}{\mathbf{c}}\right]^{\mathbf{K-1}} \mathbf{e}^{-(\mathbf{V/c})^{\mathbf{K}}} \ldots (74)$$

Here, **k** is the Weibull shape factor and **c** is scale factor. The cumulative distribution function of the velocity **V** gives us the fraction of time (or probability) that the wind velocity is equal or lower than **V**. Thus the cumulative distribution **F(V)** is the integral of the probability density function. Thus,

$$\mathrm{F}(\mathbf{V}) = \int_{\mathbf{0}}^{\infty} \mathbf{f(V)dV} = \mathbf{1} - \mathbf{e}^{-(\mathbf{V/c})^{\mathbf{K}}} \ldots (75)$$

Average wind velocity of a regime, following the Weibull distribution is given by

$$\mathbf{V_m} = \mathbf{c}\,\mathbf{\Gamma}\left[\mathbf{1} + \frac{\mathbf{1}}{\mathbf{k}}\right] \ldots (76)$$

The standard deviation of wind velocity, following the Weibull distribution is

$$\boldsymbol{\sigma}_{\mathbf{V}} = \mathbf{c}\left\{\mathbf{\Gamma}\left[\mathbf{1} + \frac{\mathbf{2}}{\mathbf{k}}\right] - \mathbf{\Gamma}^{\mathbf{2}}\left[\mathbf{1} + \frac{\mathbf{1}}{\mathbf{k}}\right]\right\}^{\mathbf{1/2}} \ldots (77)$$

The cumulative distribution function can be used for estimating the time for which wind is within a certain velocity interval. Probability of wind velocity being between $\mathbf{V_1}$ and $\mathbf{V_2}$ is given by the difference of cumulative probabilities corresponding to $\mathbf{V_2}$ and $\mathbf{V_1}$. Thus

$$\mathbf{P}(\mathbf{V_1} < \mathrm{V} < \mathbf{V_2}) = \mathbf{F}(\mathbf{V_2}) - \mathbf{F}(\mathbf{V_1}) \ldots (78)$$

$$\mathbf{P}(\mathbf{V_1} < \mathrm{V} < \mathbf{V_2}) = \mathbf{e}^{-(\mathrm{V_1/c})^{\mathbf{k}}} - \mathbf{e}^{-(\mathrm{V_2/c})^{\mathbf{k}}} \ldots (79)$$

The probability for wind exceeding $\mathbf{V_X}$ in its velocity is given by

$$\mathbf{P}(\mathbf{V} > \mathbf{V_X}) = \mathbf{1} - \left[\mathbf{1} - \mathbf{e}^{-(\mathrm{V_X/c})^{\mathbf{k}}}\right] = \mathbf{e}^{-(\mathrm{V_X/c})^{\mathbf{k}}} \ldots (80)$$

Example: A wind turbine with cut-in velocity 4 m/s and cut-out velocity 25 m/s is installed at a site with Weibull shape factor 2.4 and scale factor 9.8 m/s. For how many hours in a day, will the turbine generate power? Also estimate the probability of wind velocity to exceed 35 m/s at this site.

Solution:

$$\mathrm{P}(\mathrm{V_4} < \mathrm{V} < \mathrm{V_{25}}) = \mathrm{e}^{-(4/9.8)^{2.4}} - \mathrm{e}^{-(25/9.8)^{2.4}} = 0.89$$

Hence in a day, the turbine will generate power for 0.89 × 24 = 21.36 h

$$P = (V > V_{35}) = e^{-(35/9.8)^{2.4}} = 0.000000001$$

Thus, the chance of wind getting stronger than 35 m/s at this site is very rare.

For analyzing a wind regime following the Weibull distribution, we have to estimate the Weibull parameters **k** and **c**. The common methods for determining k and c are:
1. Graphical method. 2. Standard deviation method. 3. Moment method.
4. Maximum likelihood method, and, 5. Energy pattern factor method.

16.1.1: Standard deviation method:

The Weibull factors **k** and **c** can also be estimated from the mean and standard deviation of wind data. Once $\boldsymbol{\sigma_V}$ and $\mathbf{V_m}$ are calculated for a given data set, then **k** can be determined by solving the above expression numerically.
In a simpler approach, an acceptable approximation for **k** is

$$\mathbf{k} = \left\{\frac{\sigma_V}{V_m}\right\}^{-1.090} \quad \dots (81)$$

$$\mathbf{c} = \frac{2V_m}{\sqrt{\pi}} \quad \dots (81)$$

More accurately, **c** can be found using the expression

$$\mathbf{c} = \frac{V_m\, k^{2.6674}}{0.184 + 0.816\, k^{2.73855}} \quad \dots (82)$$

Example

Estimate the Weibull factors **k** and **c**. using the standard deviation method. Mean and standard deviation are 28.08 km/h (7.80 m/s) and 10.88 km/h (3.02 m/s) respectively. Thus

$$k = \left\{\frac{10.88}{28.08}\right\}^{-1.090} = 2.81$$

$$c = \frac{28.08 \times 2.81^{2.6674}}{0.184 + 0.816 \times 2.81^{2.73855}} = 31.6\,\frac{km}{h} = 8.78\ m/s$$

16.2: Rayleigh distribution:

A simplified case of the Weibull model can be derived, approximating **k** as **2**. This is known as the Rayleigh distribution. Taking **k = 2** in equation (76) we get

$$\mathbf{V_m} = \mathbf{c}\,\Gamma(3/2) \dots (83)$$

$$\boxed{\Gamma(3/2) = \tfrac{1}{2}\Gamma(1/2) \qquad \Gamma(1/2) = \sqrt{\pi}}$$

Evaluating the above expression and rearranging,

$$\mathbf{V_m} = \mathbf{c}\frac{1}{2}\Gamma(1/2) = \mathbf{c}\frac{1}{2}\sqrt{\pi} \xrightarrow{\text{yields}} \mathbf{c} = \frac{2V_m}{\sqrt{\pi}} \dots (84)$$

Substituting for c in equation (74), we get

$$\mathbf{f(V)} = \frac{\pi}{2}\frac{\mathbf{V}}{\mathbf{V_m^2}}\ \mathbf{e}^{-\left[\frac{\pi}{4}\left(\mathbf{V}/\mathbf{V_m}\right)^2\right]} \dots (85)$$

Similarly, the cumulative distribution is given by

$$\mathbf{F(V) = 1 - e}^{-\left[\frac{\pi}{4}\left(\mathbf{V}/\mathbf{V_m}\right)^2\right]} \dots (86)$$

Under the Rayleigh distribution, the probability of wind velocity to be between $\mathbf{V_1}$ and $\mathbf{V_2}$ is

$$\mathbf{P(V1 < V < V2) = e}^{-\left[\frac{\pi}{4}\left(\mathbf{V_1}/\mathbf{V_m}\right)^2\right]} - \mathbf{e}^{-\left[\frac{\pi}{4}\left(\mathbf{V_2}/\mathbf{V_m}\right)^2\right]} \dots (87)$$

The probability of wind to exceed a velocity of $\mathbf{V_X}$ is given by

$$\mathbf{P(V > V_X) = 1 - \left[1 - e^{-\left[\frac{\pi}{4}\left(V_X/V_m\right)^2\right]}\right] = e^{-\left[\frac{\pi}{4}\left(V_X/V_m\right)^2\right]}} \dots (88)$$

17: Energy estimation of wind regimes

17.1: Weibull based approach:

For a unit area of the rotor, power available (P_W) in the wind stream of velocity **V** is

$$\mathbf{P_W = \frac{1}{2}\rho_a V^3} \dots (89)$$

The fraction of time for which this velocity **V** prevails in the regime is given by **f (V)**. The energy per unit time contributed by **V** is $\mathbf{P_W\, f\,(V)}$. Thus the total energy, contributed by all possible velocities in the wind regime, available for unit rotor area and time may be expressed as

$$\mathbf{E_D = \int_0^\infty P_W\ f\,(V)\, dV} \dots (90)$$

Substituting for $\mathbf{P_W}$ and **f (V)** in the above expression and simplifying, we get

$$\mathbf{E_D = \frac{\rho_a\, k}{2\, c^k}\int_0^\infty V^{(k+2)}\ e^{-(V/c)^k}\, dV} \dots (91)$$

After some arrangements;

$$\mathbf{E_D = \frac{\rho_a\, C^3}{2}\frac{3}{k}\,\Gamma\left(\frac{3}{k}\right)} \dots (92)$$

Once $\mathbf{E_D}$ is known, energy available over a period ($\mathbf{E_I}$) can be calculated as

$$\mathbf{E_I = E_D T = \frac{\rho_a\, C^3 T}{2}\frac{3}{k}\,\Gamma\left(\frac{3}{k}\right)} \dots (93)$$

Where **T** is the time period. For example, **T** is taken as 24 when we calculate the energy on daily basis.

Referring to figure (14), peak of the probability density curve represents the most frequent wind velocity $\mathbf{V_F}$. For our analysis, let us rewrite the probability density function as

$$f(V) = \frac{K}{c^k}\, V^{k-1}\, e^{-(V/c)^k} \dots (94)$$

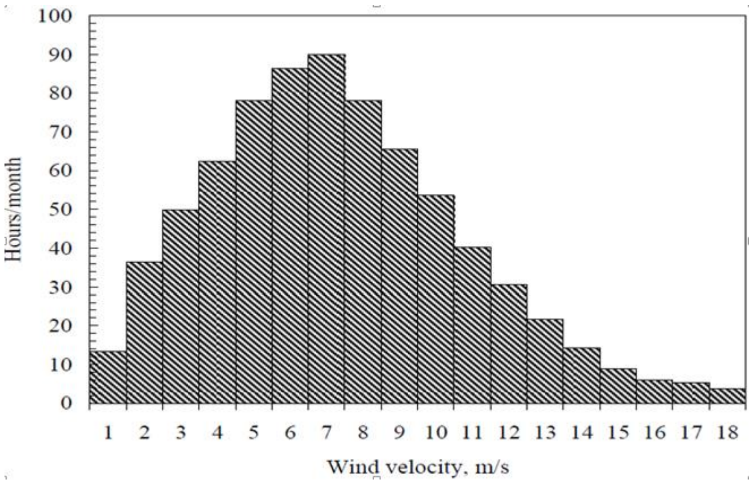

Figure (14): Probability distribution of monthly wind velocity.

For the most frequent wind velocity,

$$\mathbf{f}^{`}(\mathbf{V}) = \mathbf{0} \dots (95)$$

This means that,

$$\mathbf{V = c} \left[\frac{\mathbf{k-1}}{\mathbf{k}}\right]^{\mathbf{1/k}} \dots (96)$$

This implies that **f (V)** is maximum at **V** represented by equation (96). We will indicate the most frequent wind velocity in the regime by $V_{\mathbf{F\ Max}}$. Thus

$$\mathbf{V_{F\ Max}} = \mathbf{c} \left[\frac{\mathbf{k-1}}{\mathbf{k}}\right]^{\mathbf{1/k}} \dots (97)$$

Now let us consider the velocity contributing maximum energy to the regime ($\mathbf{V_{E\ Max}}$). Energy per unit rotor area and time, contributed by a velocity V is

$$\mathbf{E_V = P_W\, f(V)} \dots (98)$$

Substituting for $\mathbf{P_W}$ and **f (V)**, we get

$$\mathbf{E_V} = \frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{\rho}_\mathbf{a}\, \mathbf{V^3}\, \frac{\mathbf{K}}{\mathbf{c}} \left\{\frac{\mathbf{V}}{\mathbf{c}}\right\}^{\mathbf{k-1}} \mathbf{e}^{-(\mathbf{V/c})^{\mathbf{k}}} \dots (99)$$

$$\mathbf{E_V = B\, V^{(K+2)}\, e^{-(V/c)^k}} \dots (100) \quad \left[\mathbf{B} = \frac{\boldsymbol{\rho}_\mathbf{a}}{\mathbf{2}}\, \frac{\mathbf{K}}{\mathbf{c^k}}\right]$$

For $\mathbf{E_V}$ to be maximum,

$$\mathbf{E_V^{`} = 0} \ldots (101)$$

This implies that

$$\mathbf{V = \frac{c\,(k+2)^{1/k}}{k^{1/k}}} \ldots (102)$$

Hence, $\mathbf{V}$ represented by eq. (102) gives us the velocity contributing maximum energy to the regime. Representing this by $\mathbf{V_{E\,Max}}$, we have

$$\mathbf{V_{E\,Max} = c\left[\frac{k+2}{k}\right]^{1/k}} \ldots (103)$$

<u>Example:</u>

Estimate the wind energy density, annual energy intensity, most frequent wind velocity and the velocity responsible for maximum energy for the wind regime. k and c for this site are 2.24 and 7.31 m/s respectively. Take ($\rho = 1.24\ \mathrm{kg/m^3}$).

Table (4): Gamma values.

(n)	**1**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
Γ(n)	**1**	**0.9514**	**0.9182**	**0.8975**	**0.8873**	**0.8862**	**0.8935**	**0.9086**	**0.9314**	**0.9618**	**1**

<u>Solution</u>:

$$V_{F\,Max} = c\left[\frac{k-1}{k}\right]^{1/k} = 7.31 \times \left[\frac{2.24-1}{2.24}\right]^{1/2.24} = 5.6136\ \mathrm{m/s}$$

$$V_{E\,Max} = c\left[\frac{k+2}{k}\right]^{1/k} = 7.31 \times \left[\frac{2.24+2}{2.24}\right]^{1/2.24} = 9.72\ \mathrm{m/s}$$

$$E_D = \frac{\rho_a\,C^3}{2}\,\frac{3}{k}\,\Gamma\left(\frac{3}{k}\right) = \frac{1.24 \times (7.31)^3}{2}\,\frac{3}{2.24}\,\Gamma\left(\frac{3}{2.24}\right) = 0.289\ \mathrm{kw/m^2}$$

$E_I = E_D T = [0.289 \times T]$; T is taken as 365 × 24 = 8760 h.

$$E_I = 0.289 \times 8760 = 2531.64\ \mathrm{kwh/m^2}$$

<u>17.2: Rayleigh based approach</u>:

Considering Rayleigh distribution, wind energy density can be expressed as

$$\mathbf{E_D = \int_0^{\infty} P_w\, f(V)dV = \int_0^{\infty} \frac{4\,\rho_a}{4V_m^2}\, V^4\, e^{-\left\{\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right\}}\, dV} \ldots (104)$$

Introducing a constant $\mathbf{B}$ such that → $\mathbf{(B = \pi/4\ V_m^2\ )}$

The energy density error, is given by

$$\mathbf{E_D = B\,\rho_a \int_0^{\infty} V^4\, e^{(-BV^2)}\, dV \ldots}\ (105)$$

After some arrangements;

$$\mathbf{E_D = \frac{\rho_a}{2\,B^{3/2}}\,\Gamma[\tfrac{5}{2}] \ldots}\ (106)$$

This can be further evaluated as

$$\mathbf{E_D = \frac{3}{8}\,\frac{\rho_a \sqrt{\pi}}{B^{1.5}}} \ldots (107)$$

Now, replacing **B**, we get the energy density at the site as

$$\mathbf{E_D = \frac{3}{\pi}\,\rho_a\, V_m^3} \ \ldots (108)$$

From **E**

From $\mathbf{E_D}$, energy available for the unit area of the rotor, over a time period, can be estimated using the expression

$$\mathbf{E_I = E_D\, T = \frac{3}{\pi}\, T\, \rho_a\, V_m^3} \ldots (109)$$

For identifying the most frequent wind speed, the probability density function of the Rayleigh distribution is rewritten in terms of the constant **B**. Thus,

$$\mathbf{f(V) = 2BV\, e^{-(BV^2)}} \ldots (110)$$

Now, the condition of maxima is applied to the above expression. This yield

$$\mathbf{f^{`}(V) = 0}$$

$$\mathbf{2B\, e^{-(BV^2)}\,(1 - 2BV^2) = 0} \ldots (111)$$

Solving equation (111) for **V**, we get

$$\mathbf{V = \frac{1}{\sqrt{2B}}} \ \ldots (112)$$

Hence, **f (V)** is maximum at V represented by equation (112). Let us represent this by $\mathbf{V_{F\,Max}}$. Thus, we have

$$\mathbf{V_{F\,Max} = \frac{1}{\sqrt{2B}} = \sqrt{2/\pi}\; V_m} \ldots (113)$$

Now, let us identify the velocity contributing maximum energy to the regime. Following the Rayleigh model, the energy corresponding to a wind velocity **V**, over unit rotor area and time can be expressed as

$$\mathbf{E_V = P_w\, f(V) = B\rho_a\, V^4\, e^{(-BV^2)}} \ldots (114)$$

For maximum value of $\mathbf{E_V}$,

$$\mathbf{E_V^{`} = 0} \quad \ldots (115)$$

This implies that

$$\mathbf{B\rho_a e^{(-BV^2)}\{4V^3 + V^4(-2BV)\} = 0} \ldots (116)$$

Solving the above expression for **V**, we get

$$\mathbf{V} = \sqrt{\mathbf{2/}\boldsymbol{B}} \quad \ldots (117)$$

Hence, **Ev** has the maximum value for **V** represented by equation (117). Thus,

$$\mathbf{V_{E\,Max}} = \sqrt{\mathbf{2/}\boldsymbol{B}} = \mathbf{2}\sqrt{\mathbf{2/}\boldsymbol{\pi}}\ \mathbf{V_m} \quad \ldots (118)$$

Example:

Monthly wind velocity at a location is given in Table (5) below. Calculate the wind energy density, monthly energy availability, most frequent wind velocity and the velocity corresponding to the maximum energy.

Table (5)**:** Monthly average wind velocity.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
9.14	8.3	7.38	7.29	10.1	11.1	11.4	11.1	10.3	7.11	6.74	8.58

Solution:

Rayleigh analysis under the SITE module is used here.

Table (6)**:** Wind energy potential based on the Rayleigh analysis.

Month	E_D (kW/m^2)	E_I (kW/m^2/month)	V_{Fmax} (m/s)	V_E (m/s)
January	0.90	666.95	7.29	14.58
February	0.67	451.11	6.62	13.24
March	0.47	351.09	5.89	11.77
April	0.46	327.49	5.82	11.63
May	1.20	889.30	8.03	16.05
June	1.59	1146.72	8.83	17.66
July	1.76	1307.78	9.13	18.25
August	1.59	1184.94	8.83	17.66
September	1.29	931.78	8.24	16.48
October	0.42	313.95	5.67	11.34
November	0.36	258.82	5.38	10.75
December	0.74	551.72	6.84	13.69

18: Energy estimation of wind turbines

To estimate the energy generated by the turbine at a given site over a period, the power characteristics of the turbine is to be integrated with the probabilities of different wind velocities expected at the site. For example, power curve of a wind turbine is shown in Fig. 15a whereas Fig. 15b presents the Weibull probability density function of a candidate site.

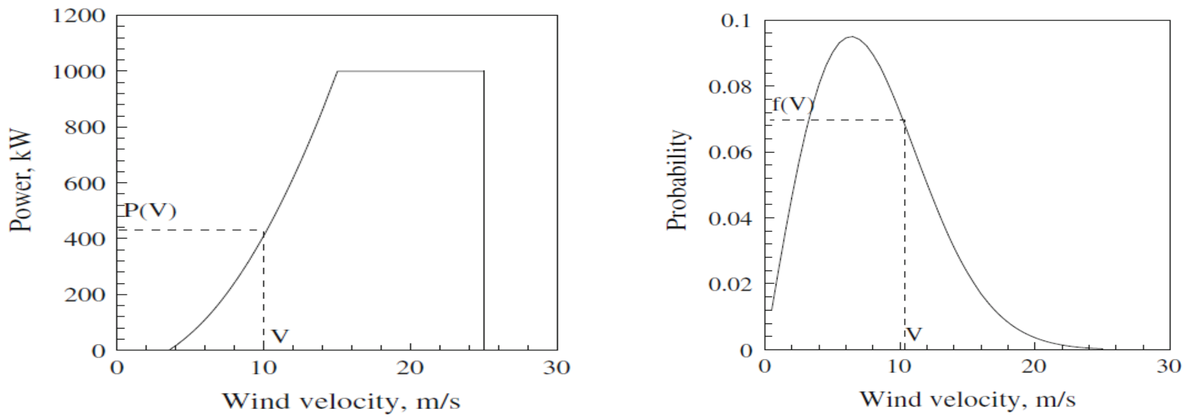

Fig (15): (a) power curve of 1 MW wind turbine, (b) probability density curve.

The fraction of energy contributed by any wind velocity **V** is the product of power corresponding to **V** in the power curve, that is **P**(**V**) and the probability of **V** in the probability density curve, which is **f**(**V**). Thus, at this site, the total energy generated by the turbine E, over a period T, can be estimated by

$$\mathbf{E} = \mathbf{T} \int_{V_i}^{V_o} \boldsymbol{P_V}\ f(V)\ dV \ \ldots (119)$$

The power curve has two distinct productive regions from $\boldsymbol{V_I}$ to $\boldsymbol{V_R}$ and $\boldsymbol{V_R}$ to $\boldsymbol{V_O}$. Thus, let us take

$$\mathbf{E} = \boldsymbol{E_{IR}} + \boldsymbol{E_{RO}} \ \ldots (120)$$

$$\boldsymbol{E_{IR}} = \mathbf{T} \int_{V_I}^{V_R} \boldsymbol{P_V}\ f(V)\ dV \ \ldots (121)$$

$$\boldsymbol{E_{RO}} = \boldsymbol{T}\ \boldsymbol{P_R} \int_{V_R}^{V_O} f(V)\ dV \ \ldots (123)$$

So,

$$E_{IR} = P_R\ \mathrm{T} \int_{V_I}^{V_R} \left[\frac{V^n - V_I^n}{V_R^n - V_I^n}\right] \frac{k}{c} \left(\frac{V}{c}\right)^{k-1} e^{-(V/c)^k} dV$$

$$\rightarrow E_{IR} = \left(\frac{P_R\ \mathrm{T}}{V_R^n - V_I^n}\right) \int_{V_I}^{V_R} [V^n - V_I^n]\ \frac{k}{c} \left(\frac{V}{c}\right)^{k-1} e^{-(V/c)^k} dV \ \ldots (124)$$

Let us introduce the variable **X** such that

$$\mathrm{X} = (V/c)^k\ , \qquad dX = (k/c)(V/c)^{k-1}\ , and\ \ V = cX^{(1/k)}$$

So,

$$\mathrm{X}_I = (V_I/c)^k\ \ , \mathrm{X}_R = (V_R/c)^k\ \ , and\ \ \mathrm{X}_o = (V_o/c)^k$$

With this substitution, and simplification thereafter, E_{IR} can be expressed as

$$E_{IR} = \left(\frac{P_R\ \mathrm{T}\ c^n}{V_R^n - V_I^n}\right) \int_{X_I}^{X_R} X^{n/k}\ e^{-X} dX - \left(\frac{P_R\ \mathrm{T}\ V_I^n}{V_R^n - V_I^n}\right) [e^{-X_I} - e^{-X_R}] \ \ldots (125)$$

For the second performance region, $\boldsymbol{E_{RO}}$ may be represented as

$$\boldsymbol{E_{RO}} = \boldsymbol{T\, P_R} \int_{V_R}^{V_O} \frac{\mathbf{K}}{\mathbf{c}} \left[\frac{\mathbf{V}}{\mathbf{c}}\right]^{\mathbf{K-1}} \mathbf{e}^{-(\mathbf{V}/\mathbf{c})^{\mathbf{K}}}\, dV \;\rightarrow \boldsymbol{E_{RO}} = \boldsymbol{T\, P_R}\left(e^{-X_R} - e^{-X_O}\right) \ldots (126)$$

The capacity factor $\boldsymbol{C_F}$ which is the ratio of the energy actually produced by the turbine to the energy that could have been produced by it, if the machine would have operated at its rated power throughout the time period is given by

$$C_F = \frac{E}{P_R\, T} = \frac{E_{IR}+E_{RO}}{P_R\, T} \ldots (127)$$

Thus,

$$C_F = \frac{c^n}{V_R^n - V_I^n} \int_{X_I}^{X_R} X^{n/k}\, e^{-X} dX - \left(\frac{V_I^n}{V_R^n - V_I^n}\right)\left[e^{-X_I} - e^{-X_R}\right] + \left(e^{-X_R} - e^{-X_O}\right) \ldots (128)$$